אפקה
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סמינר מדעי היסוד

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Power laws, sandpiles, and pseudo-inverses

תקציר

Power laws are ubiquitous in nature and society. For instance:
* Magnitudes and frequencies of earthquakes are related by a power law (Gutenberg-Richter law),
* The metabolic rate of an animal and its mass are related by a power law (Kleiber's law).
* The growth of cities of follows a power law (Gibrat's law).

Various generative models, such as the preferential attachment model of Barabási et al. have been proposed in order to account for this strange efficiency of power laws. In this talk I will talk about another model, the so called abelian sandpile model, proposed by Bak, Tang, and Wiesenfeld in 1987.

Imagine a large chess board, with a few grains of sand on each square, arranged in a tiny tower. Every unit of time a grain is dropped on a random square; if the square now has 4 grains, it ``topples'' and emits a grain to each of its four neighbors - and so forth. Grains emitted by squares on the boundary just fall off the board.

Now, most of the time, the addition of a single grain produces no or little effect. But every once in a while, a single grain will trigger an avalanche. The durations of the avalanches will then be distributed according to a power law.

In my talk I will discuss a mathematical variant of this model, the chip-firing game, originally introduced by Björner, Lovász and Shor in 1991. In the game the grid is replaced by a graph with n vertices and the grains by N poker chips. The random choice is replaced by a solitaire player's discretion.

They have proved the remarkable result that whenever this game terminates, it always does so in the same number of moves, irrespective of gameplay! (I will explain the background for this seemingly shocking result). They also gave an upper bound on the duration of the game (i.e. the number of moves), using a pseudo-inverse.

However, I have observed that the game usually ends in far fewer moves than their bound.

To explain this phenomenon, I will show how their analysis can be substantially refined, yielding greatly improved bounds for some graph classes. For example, for strongly regular graphs the Björner, Lovász and Shor bound is $O(nN)$ whereas my new bound is $O(n + N)$.

No prior knowledge is assumed. As a bonus, I will show a “proof” that all but the tiniest animals must have lungs or gills.

מתאמים: פרופ' י. גולדמן, ד"ר ש. מיברג, פרופ' י. סטאנצ'סקו ופרופ' ד. פישלוב

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